

# Liquidity Preference and Financial Intermediation

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We examine the characteristics of optimal monetary policies in a general equilibrium model with incomplete markets. Markets are incomplete because of uninsured preference uncertainty, and because productive capital is traded infrequently. Rational individuals are willing to hold a liquid asset—"money"—at a premium. Monetary policy interacts with existing financial institutions to determine this premium, as well as the level of precautionary holdings. We show that inflation is expansionary, and that the optimal inflation rate is positive if there is no operative banking system (the Tobin effect). Otherwise, efficiency requires that money be undominated in its rate of return (the Friedman Rule).

## 1. INTRODUCTION

We analyse the preference for liquidity and evaluate the effects of monetary policy in a framework where individuals optimize, and goods and asset markets are competitive. Liquidity needs of individuals, and characteristics of assets, are consequences of the structure of preferences, technology and information. Equilibrium is compatible with differences in the rates of return on assets differing in liquidity; with divergences between household savings and aggregate investment in productive capital; and distortion of household consumption plans. We examine how these quantities are determined in alternative financial environments, the extent to which monetary or interest rate policies are effective and desirable, and the relative efficiency of alternative instruments which serve private liquidity needs.

We find that monetary policy is effective, and inflation is expansionary, if the consumption plans of individuals are constrained by the lack of liquidity. At the same time, such expansion can be welfare inferior. Intermediation, which allows liquid assets to be issued against productive investment, is both feasible and desirable. Unlike inflation, intermediation achieves welfare enhancing expansion. This allows a comparison of the Tobin effect (Tobin (1965)) and the Friedman rule (Friedman (1969)) in a general equilibrium model. Crudely speaking, the first indicates that inflation is desirable, while the other claims that deflation is. The difference, as we show, depends on the degree of intermediation.

Money and productive capital are simultaneously held by individuals, even when money has a lower rate of return. This is due to the fact that money is "liquid"—it can be used to pay for goods at any time and in all states of nature. Capital, on the other

hand, is traded infrequently because its worth is not observed sufficiently often. Individuals value liquidity because of uncertainty about future consumption needs; in our model, this is represented by uninsured preference uncertainty as in Diamond and Dybvig (1983).

We can think of "money" as any asset that meets the liquidity needs of individuals. Its rate of return may be controlled by monetary policy. The nature of this asset has substantial effects on output, on welfare, and on the qualitative properties of optimal monetary policy. Fiat money does not bear interest; its rate of return is the negative of the inflation rate, which is controlled by variations in the rate of growth of money supply. Bank money, such as demand deposits, is backed by productive investment, which finances the payment of interest. Intermediate forms of money can be identified with restrictions on the reserve ratio, which determines the extent to which money is backed, and its interest payments.

A change in the rate of return on money has real effects, as money and physical capital are substitute assets for households. Monetary policy is effective because a fall in the rate of return on money makes investment in capital more attractive, leading to greater investment and output. This is precisely the Tobin effect. At the same time, the undesirability of expansionary policy is simple to understand. If money is held for a purpose, this purpose is badly served by an increase in its opportunity cost. Any increase in the cost of liquidity reduces consumers' welfare. In sum, the Tobin effect is expansionary, but directly welfare-inferior. With fiat money, inflation is sustained by money growth. The desirability of these cash transfers counteracts the direct effect to some extent. The optimal rate of inflation may thus be positive, though typically small.

Liquid assets are held for a purpose, and efficiency requires that its cost be made as small as possible. This, of course, is the Friedman rule, which suggests that money pay the same rate of interest as capital. This is true provided money is backed, so that interest payments on money are financed by productive investment: intermediation allows money to earn the same, or even higher return than capital, and achieves an increase in output as well as in welfare. This can be interpreted as the outcome of a system where all money is bank money, and the deposit interest rate is no less than the return which could be earned elsewhere. At this solution individuals hold their entire savings in the form of bank deposits, all investment is mediated by the banking sector, and banks make zero profits. In sum, the optimal quantity of money is rather different if we think of M2, rather than M0, as the operative notion of money.

Clearly, the optimal quantity of money depends on the motives for holding it, as well as the technology for producing it. It is standard, especially in recent literature, to view money primarily as a medium of exchange, with the private sector facing a cash-in-advance constraint, as in Grossman and Weiss (1983), or Lucas and Stokey (1987). In contrast, we study an economy where money is held simultaneously as a store of value. At the same time, we assume that the provision of inside money is socially costless. Such costs, when present, must be accounted for in determining the efficient provision of liquidity. A series of recent papers (Gillman (1993), Ireland (1994), and Aiyagari and Eckstein (1994)) study the effect of costly credit on monetary policy. Typically, optimal inflation rates decline with the cost of credit. Lower inflation encourages individuals to hold fiat money, which is produced at zero cost. Interestingly, Ireland (1994) concludes that bank money is likely to replace fiat money in a growing economy, if the relative cost of credit falls with output growth. We consider optimal interest rate policies in an economy where this transition has been completed.

The paper is organized as follows. In Section 2, we describe the economy in terms of preferences, technology, and information. Section 3 analyses the portfolio choice and

consumption patterns of individuals subject to liquidity constraints. In Section 4, we consider the role of policy, and optimal policy choices, in different financial environments. Section 5 discusses issues in decentralization of outcomes with intermediation. Section 6 concludes. All proofs are reported in the Appendix.

## 2. THE ECONOMY: PREFERENCES, TECHNOLOGY, AND INFORMATION

There are overlapping generations, each consisting of a continuum of individuals who live for three periods. They are born in period 0, and consume only in periods 1 and 2. There is only one commodity in each period, which can be consumed or invested. Every individual is endowed with  $e$  units of this commodity when she is born; they have no further endowments.

In period 0, each individual chooses her investment portfolio, allocating initial wealth across alternative stores of value. When making this choice, she does not know her own preferences in the future. We model this as a preference shock which realizes in the middle period: the uncertainty about preferences is captured by the parameter  $\theta \in [0, 1]$ . A consumer with preference parameter  $\theta$  has utility function

$$U(c_1, c_2; \theta) = \theta \ln(c_1) + (1 - \theta) \ln(c_2); \quad 0 < \theta \leq 1,$$

where  $c_1, c_2$  are consumption levels in periods 1 and 2. A consumer with  $\theta$  close to 0 is extremely patient, and willing to postpone consumption, and one with  $\theta$  close to 1 is similarly impatient. We refer to  $\theta$  as the degree of impatience. Alternately, a high value of  $\theta$  captures the possibility that some households may have early, and unforeseen, consumption needs.

Faced with preference uncertainty in period 0, individuals maximize expected utility. They evaluate uncertainty in terms of a common, objective probability distribution. This distribution  $F$  of preferences within each generation is constant over time, and we assume  $F$  to be uniform on  $[0, 1]$

$$\text{Prob}(\theta \leq x) = F(x) = x \quad \text{for } 0 < x \leq 1.$$

Each individual may invest all or part of her endowment in a technology. This technology has a single input, constant returns to scale, and takes two periods to yield output. The production function is

$$y_t = (1 + \rho)I_{t-2}; \quad \rho > 0,$$

with  $y_t$  as output in period  $t$ , and  $I_t$  the amount invested at  $t$ . Importantly, this technology is universal, so that every individual has access to it. We assume two further characteristics, which are crucial in generating the need for liquidity. Investment  $I_t$  is assumed to be both irreversible and unobservable in period  $t + 1$ . Irreversibility implies that capital cannot be pulled out in the middle period. In addition, the quantity of capital currently invested in a project cannot be observed by an outsider. At any time  $t$ , a project with positive  $I_{t-1}$  is observationally equivalent to one with  $I_{t-1} = 0$ . The latter can be produced costlessly, implying that the former cannot possibly command a positive price. This results in a collapse of secondary capital markets, for reasons similar to Akerlof (1970).

We could think of  $I$  as seed corn, which takes two periods to yield its crop. Investment is irreversible, so that seed corn cannot be pulled out and consumed in the middle period. Unsown land is freely available, and of zero value. The quantity of seed cannot be observed after sowing. As a consequence, the market for sown land collapses.

We consider environments where individuals can hold a liquid asset in addition to physical capital. This asset can be traded in every period and in each state of nature. It has risk-free return  $r_t$  in period  $t$ . The alternative financial environments correspond to this asset being government bonds (Diamond (1965)), fiat money (Tobin (1965)), or bank money (Ireland (1994)).

### 3. PORTFOLIO CHOICE

We consider the portfolio choice problem facing a typical consumer in any period. The consumer has initial endowment  $e$ , and may receive transfers  $\tau$  in the initial period, yielding total wealth  $w = e + \tau$ .

In period 0, she chooses a portfolio  $(m, I)$  with  $m + I = w$ , where  $I$  is the amount of physical capital, and  $m$  refers to real money holdings. A unit of physical capital yields nothing in one period, and  $1 + \rho$  in two periods. A unit of money yields  $1 + r$  in one period.<sup>1</sup>

In period 1, the preference shock  $\theta$  realizes, and the consumer chooses first-period consumption  $c_1$ , subject to the liquidity constraint  $c_1 \leq (1 + r)m$ . Consumption in the second period is financed by the yield of investment plus any unspent money balances,  $c_2 = (1 + \rho)I + (1 + r)((1 + r)m - c_1)$ .

The consumer's problem (CP) can be written as

$$\max_{m, c_1(\theta)} \int_0^1 [\theta \ln(c_1(\theta)) + (1 - \theta) \ln((1 + \rho)(w - m) + (1 + r)((1 + r)m - c_1(\theta)))] d\theta,$$

subject to

$$0 \leq m \leq w,$$

and

$$c_1(\theta) \leq (1 + r)m \quad \text{for } \theta \in [0, 1].$$

We characterize the solution to this problem in Theorem 1.

**Theorem 1.** *The problem (CP) has a unique solution. Let  $\gamma = 1 - (1 + r)^2 / (1 + \rho)$  be the proportional liquidity premium. For each  $w > 0$ :*

1. *The optimal portfolio  $(m, I)$  is  $m = \mu(\gamma)w$ ,  $I = (1 - \mu(\gamma))w$ , where*

$$\mu(\gamma) = \begin{cases} ((1 + 2\gamma) - \sqrt{1 + 4\gamma - 4\gamma^2}) / 4\gamma^2 & \text{if } 0 < \gamma < 1; \\ 1 & \text{if } \gamma \leq 0; \end{cases}$$

2. *Let  $\tilde{c}_1(\theta) = \theta(\omega - \gamma m)(1 + \rho) / (1 + r)$ . For each  $\theta \in [0, 1]$ , period 1 consumption is*

$$c_1(\theta) = \min[\tilde{c}_1(\theta), m(1 + r)].$$

The formal proof is reported in the Appendix. The argument is constructed as follows. If  $\gamma \leq 0$ , money dominates capital in its rate of return, so that individuals hold their entire wealth in money and their consumption levels are unconstrained by liquidity. On the other hand, if  $\gamma > 0$ , individuals face a tradeoff between the higher yield on investment and the

1. In principle, the rate of return on money may vary over time. Assuming it to be constant simplifies presentation, and corresponds to a stationary equilibrium path.

liquidity services of money. Given any choice of  $m$ , consumption  $c_1$  cannot exceed  $m(1+r)$ . For  $m < w$  this constraint binds whenever  $\theta$  is relatively large: for these realizations the individual will be unable to finance her preferred consumption pattern, while for low values of  $\theta$  she will end up with money holdings in excess of her preferred period 1 consumption. In choosing  $m$ , she trades off these two effects.

Some properties of the solution are of interest.

**Property 1.** *The demand for liquidity,  $m = w\mu(\gamma)$ , is strictly decreasing in  $\gamma \in (0, 1)$ . Further,  $\mu(1) = \frac{1}{2}$ , and  $\mu(0) = 1$ .*

We know, from standard portfolio theory, that asset demands respond positively to own rates of return if the degree of relative risk-aversion is not too high. This is reflected here in the monotonic behaviour of money demand. In general, strictly positive money balances will be held whenever  $c_1$  is a necessary good. In our example, the fraction of wealth held in money is at least 50%—this reflects the symmetry of the uniform distribution. As the liquidity premium falls, the proportion of wealth held in money rises. If money is undominated in its rate of return, all wealth is held in money.

**Property 2.** *For each  $\gamma \in (0, 1)$ , the proportion of consumers who are constrained by liquidity is*

$$\lambda(\gamma) = 2\gamma\mu(\gamma).$$

*This proportion is increasing in  $\gamma$ , with  $\lambda(0) = 0$  and  $\lambda(1) = 1$ . Further,  $\lambda(\gamma) = 0$  whenever  $\gamma \leq 0$ .*

The incidence of liquidity constraints is a consequence of portfolio choice. Consumers are liquidity constrained whenever  $\theta$  is sufficiently large. For money holding  $m$ , the threshold value  $\theta_*$  is such that  $\tilde{c}_1(\theta_*) = m(1+r)$ . The fraction of the population so constrained is simply  $1 - \theta_*$ , given by  $\lambda(\gamma)$  above. As the liquidity premium increases, consumers choose to hold less money and face the prospect of binding liquidity constraints more often.

Theorem 2 reports the value function: this is a consequence of Theorem 1. The value function is useful in evaluating monetary policies.

**Theorem 2.** *Let  $\gamma$  and  $\mu(\gamma)$  be as in Theorem 1. The expected utility of a consumer in period 0 is*

$$V(r; w) = \begin{cases} \ln w + \frac{1}{2} \ln(1+r)\mu(1-\gamma\mu) - \frac{1}{2}(1-2\gamma\mu) & \text{for } 0 < \gamma < 1; \\ \ln w + \frac{3}{2} \ln(1+r) - \frac{1}{2} & \text{for } \gamma \leq 0. \end{cases}$$

Property 3 shows that consumers' welfare is increasing in  $r$ , the rate of return on money; here, the income and liquidity effects operate in the same direction.

**Property 3.** *An increase in  $r$  makes the consumer better off, ex ante.*

Properties 1 and 3 have an important consequence for the desirability of expansionary policies. The direct welfare effects of interest rate changes are in a direction opposite to output effects. Specifically, an increase in  $\gamma$ , the opportunity cost of holding money, is welfare inferior (Property 3); it is expansionary because a larger proportion of private

wealth is held in the form of productive capital (Property 1). At the same time, Property 2 identifies an observable implication of this inefficiency: the lower level of money holdings distorts the consumption plans of a greater proportion of the population. Property 3 also shows that financial policies that reduce the premium on liquidity have the potential to improve welfare. The availability of such policies depends on the nature of the liquid asset  $m$ . In the next section, we consider alternative financial structures with this problem in mind.

#### 4. WELFARE, EXPANSION, AND FINANCIAL STRUCTURES

Individuals are willing to hold a dominated asset, such as money, because of its liquidity services. The interest rates affects welfare and the aggregate impact of liquidity constraints. It is then natural to ask what outcomes can arise at equilibrium, and how alternative financial policies affect these outcomes.

The answer depends very much on the nature of financial assets and institutions available. We consider alternative financial environments, restricting our analysis to stationary competitive equilibria.

Money—more correctly, the liquid asset  $m$ —has three characteristics of interest, which we refer to as  $r$ ,  $\tau$ , and  $\Delta$ . The first is its rate of return  $r$ . This is an outcome of competitive trading on financial markets, and may be affected by policy choices made by monetary authorities. We consider two instruments of monetary policy.

Monetary authorities can use transfers  $\tau$  to control the quantity of money. This corresponds to growth in real balances, positive or negative. We assume, for simplicity, that all such transfers are made to the young and are constant across households. Feasibility requires  $\tau > -e$ , so that taxes do not exceed initial endowments.

Money may also be backed by productive investment,  $I_*$ , which finances interest payments on money.  $I_*$  is the amount of capital invested by the banking sector, and corresponds to the reserve ratio  $\Delta = 1 - I_*/m$ , with  $\Delta \in [0, 1]$ .

These three characteristics ( $r$ ,  $\tau$ ,  $\Delta$ ) cannot be independent if goods and money markets clear at each time. A consumer with initial wealth  $w = e + \tau$  chooses a portfolio  $m$ ,  $I = w - m$  in response to interest rates  $r$ . Aggregate consumption levels are

$$\bar{C}_i = \int_0^1 c_i(\theta) d\theta; \quad i = 1, 2.$$

The quantity of institutional investment is  $I_* = (1 - \Delta)m$ . Aggregate investment is the sum of household and institutional investments:  $\bar{I} = I + I_* = w - m\Delta$ . Goods markets clear in each period if

$$(E) \quad \bar{C}_1 + \bar{C}_2 + \bar{I} = e + \bar{I}(1 + \rho).$$

The quantities  $m$ ,  $\bar{C}_1$ ,  $\bar{C}_2$  are all proportional to  $w = e + \tau$ , so that the restriction (E) can be represented by a function  $\eta$

$$(E) \Leftrightarrow \eta(r, \tau, \Delta) = 0.$$

Along with the feasibility constraints

$$\tau > -e; \quad r > -1; \quad \Delta \in [0, 1],$$

this summarizes the restrictions on monetary policy imposed by stationarity and market clearing. The financial environments considered below can be seen as alternative, and additional restrictions on the characteristics of money.

Consider the restriction  $\Delta = 1$ . This corresponds to  $m$  being cash or unbacked fiat money. The quantity of money is controlled by monetary authorities. Changes in cash transfers  $\tau$  affect the growth of money supply and the inflation rate,  $\pi = r/(1+r)$ .

The additional restriction  $\tau = 0$  can be understood, either as  $m$  being unbacked public debt, or as fiat money with the supply of money held constant over time.

If, on the other hand, we impose the restriction  $\tau = 0$ , and allow reserves  $\Delta$  to vary between 0 and 1, the corresponding asset is bank money. Changes in  $\Delta$  affect  $r$ , the deposit interest rate. Stationary equilibria with intermediation correspond to  $(r, \Delta)$  which clear capital markets every period.

Monetary authorities can choose financial policies to maximize welfare, subject to the constraint that interest rates clear capital markets. The objective function is  $V(r, w = e + \tau)$ , and the constraint is that  $\eta(r, \tau, \Delta) = 0$ . In general, this constraint is not well-behaved in the programming sense: convexity or even monotonicity are not satisfied everywhere. We consider restricted versions of the problem, corresponding to  $\Delta = 1$  or  $\tau = 0$ , and report a numerical solution to the general problem.

#### 4.1. Public debt

We first consider the outcome without any transfers or financial intermediation. This corresponds to an economy where public debt services private liquidity needs, and current borrowing is used to repay past debts. Theorem 3 shows that the interest rate on liquid assets is zero at the unique stationary equilibrium.

**Theorem 3.** *Let  $\Delta = 1$  and  $\tau = 0$ . The unique stationary equilibrium has the following characteristics:*

1. *The equilibrium rate of interest is zero:  $r = 0$ . The equilibrium liquidity premium is  $\gamma = \rho/(1 + \rho)$ ;*
2. *Aggregate investment is positive, and less than initial endowments:  $\bar{I} = I$ ,  $0 < I < e$ ;*
3. *A strictly positive proportion of consumers are liquidity constrained:  $\lambda(\rho/(1 + \rho)) > 0$ .*

Note that there are no risks associated with production here so that  $\gamma$  is a pure liquidity premium. The fact that the liquidity premium is determined by technology alone is of interest relative to the risk-free rate puzzle (Mehra and Prescott (1985), Weil (1989)). If risk-free bonds provide liquidity services, their return can be low relative to that of productive capital, irrespective of time-preferences and risk-aversion of individuals.

A zero rate of interest at the stationary equilibrium holds for zero population growth. With positive growth in population, say  $g > 0$ , the stationary interest rate is  $r = g$ , and the liquidity premium is correspondingly lower. From Property 3, an increase in  $g$  makes individuals strictly better off, a result which contrasts with the neoclassical growth model of Diamond (1965).

#### 4.2. Fiat money

The fact that money is held for a precautionary motive allows a role for active monetary policy. Monetary authorities can choose an inflation rate  $\pi = -r/(1+r)$ , and sustain this with lump sum transfers of value  $\tau$ . The expansionary effect of inflation comes from two sources. Inflation lowers the rate of return on money: in response, individuals hold less

money and invest a higher proportion of their wealth. The second effect comes from the fact that transfers may redistribute income to individuals who have higher propensities to save and invest. Aggregate output, defined as  $Y = e + (1 + \rho)(e + \tau)(1 - \mu(r))$ , is increasing in  $\tau$  and decreasing in  $r$ . But at the same time, higher inflation causes greater distortion of consumption plans, and forces liquidity constraints to bind on a greater proportion of consumers. The optimal inflation rate trades off the benefits of transfers  $\tau$  with the welfare cost of lower  $r$ ; it obtains as the solution to the following problem

$$\text{MP(1)} \quad \max_{r, \tau} V(r; e + \tau),$$

subject to the constraints

$$\eta(r, \tau, 1) = 0; \quad r > -1; \quad \rho > -e.$$

Theorem 4 establishes that, with fiat money, an optimal inflation rate exists. For this and subsequent theorems, we define  $(1 + r^*) = \sqrt{1 + \rho}$ . Values of  $r < r^*$  correspond to positive liquidity premia.

**Theorem 4 (The Tobin Effect).** *Let  $\Delta = 1$ . For each  $\tau \in (-1 - (1/(1 + 1.5r^* + 0.5r^{*2})))e, e$ , there exists a unique stationary equilibrium with real interest rate  $r(\tau)$  and inflation rate  $\pi(\tau) = -r(\tau)/(1 + r(\tau))$ . It has the following properties:*

1.  $r(\tau)$  is strictly decreasing and  $\pi(\tau)$  is strictly increasing in  $\tau$ ;
2. Inflation is expansionary: an increase in  $\tau$  increases aggregate output;
3. An optimal inflation rate,  $\hat{\pi}$ , exists and is finite. The liquidity premium is positive at this optimum;
4. Zero inflation is suboptimal: an increase in  $\tau$  from  $\tau = 0$  is welfare-increasing.

#### Remarks.

- Theorem 4 establishes that an interior solution to the problem MP(1) exists, and also that  $(dV/d\tau)|_{\tau=0} > 0$ . This clearly establishes a role for active monetary policy. The argument also suggests that  $\hat{\pi} > 0$ . Numerical simulations for values of  $\rho \in (0, 1]$  establish that this is indeed true, and that the optimal inflation rate is increasing in  $\rho$ . Unfortunately, these properties do not appear to be analytically tractable.
- Clearly, the optimal inflation rate is sensitive to perturbations in the parameters. It seems likely that the optimal rate will decrease with the degree of risk-aversion, because the marginal value of transfers is lower. Our example has the property that the demand for money is monotonic in  $r$ , so that the Tobin effect is always positive. The effect could be perverse if individuals are more risk-averse. Evidence from alternative sources suggest that risk-aversion is indeed high enough (Mehra and Prescott (1985), for example). This would imply that inflation has non-monotone effects on output. Arguments similar to ours suggest that the optimal inflation rate would be below the rate which achieves highest output.
- The Friedman rule (Friedman (1969)) on the optimal quantity of money prescribes that money should have the same rate of return as capital. This requires that  $\pi$  and  $\tau$  be negative whenever  $\rho$  is positive. The welfare-reducing effect of lump-sum taxes  $-\tau$  dominates the benefits of lower inflation, so that the rule is feasible, but undesirable in this context.



4.3. Bank money

The welfare costs of inflation are due entirely to the fact that inflation raises the liquidity premium, and distorts consumption plans. The impact of liquidity constraints on consumption can be softened by lowering this premium. We know that consumers are better off whenever the rate of return on money,  $r$ , is increased. It is possible to finance a higher rate of return on money by means other than distortionary taxation?

It is indeed possible, in the presence of intermediation. Theorem 3 showed that  $r > 0$  is incompatible with  $I_* = 0$  and  $\tau = 0$ . We show next that the solution corresponding to the Friedman rule can be attained with  $I_* > 0$ . The financial asset,  $m$ , should now be thought of as bank deposits. It is backed by the bank's capital holdings, so that  $I_* > 0$ . Importantly, inside money need not be fully backed, with occurs whenever  $I_* < m$ . The degree of intermediation is measured by  $\delta = I_*/(I_* + I)$ , the proportion of total investment mediated by financial institutions. Complete intermediation is achieved whenever  $I = 0$ , and  $\delta = 1$ .

Theorem 5 demonstrates the existence of at least one solution with complete intermediation, which achieves  $\gamma = 0$ . As there is no premium for holding liquid financial assets, individuals choose to hold all their savings in financial rather than physical assets. Consumption is not subject to liquidity constraints.

**Theorem 5 (The Friedman Rule).** *Let  $\tau = 0$ . A stationary solution with complete intermediation exists, and has the following properties:*

1. The liquidity premium is zero:  $r = r^*$ ,  $\gamma = 0$ ;
2. All investment is institutional:  $I(r^*; e) = 0$  and  $\delta = 1$ ;
3. Liquidity constraints do not bind:  $\lambda(0) = 0$ ;
4. Aggregate reserves are positive:  $\Delta = (\rho - r^*)/2\rho > 0$ .

This solution—the exact Friedman Rule—is of interest for several reasons. It corresponds to a zero-profit condition on intermediation. We show later that this outcome is in fact achieved with competitive, privately held banks. However, Theorem 5 shows only that the Friedman Rule is feasible: it may not be efficient in the presence of intermediation. The Rule is achieved with positive reserves, which are likely to be inefficient in a world with no aggregate risk. The efficient solution with intermediation solves the following problem

$$\text{MP(2)} \quad \max_{r > -1} V(r, e),$$

subject to

$$\eta(r, 0, \Delta) = 0; \quad 0 \leq \Delta \leq 1.$$

Theorem 6 shows that the solution to the problem MP(2) is different from the exact Friedman Rule; indeed, it is achieved by paying a strictly higher return on money.<sup>2</sup>

2. Interest rates greater than  $\rho$ , the certain return on capital, are feasible for the following reason. At any time, banks receive deposits, equal to  $m$ , and income  $I_*(1 + \rho)$  from previous investment. Total withdrawals are  $\bar{C}_1 + \bar{C}_2 - (1 + \rho)I$ . For each  $r \geq r^*$ , we have  $m = e$  and  $I = 0$ , so that withdrawals every period are less than consumers' total lifetime budgets:  $\bar{C}_1 + \bar{C}_2 < \bar{C}_1(1 + r) + \bar{C}_2$  whenever  $r > 0$ . It follows that reserves are positive at  $r = r^*$ , and that this surplus can be invested to pay a higher rate of return on savings.

**Theorem 6** (Efficient Intermediation). *Let  $\tau=0$ . An optimal interest rate exists and is equal to  $\bar{r}=(\sqrt{9+8\rho}-3)/2$ . At this interest rate:*

1. *The liquidity premium is negative:  $\gamma < 0$ ;*
2. *All investment is institutional:  $I=0$ , and  $\delta=1$ ;*
3. *Reserves are zero:  $I_*=e$  and  $\Delta=0$ ;*
4. *Aggregate output is  $Y=e(2+\rho)$ .*

*Remarks.*

- Consumers are better off with the highest possible return on money. The maximal feasible interest rate is  $\bar{r}$ , achieved with zero reserves. In the presence of aggregate uncertainty, optimal financial policies may imply positive reserves as well as state-dependent return on money. The restriction that money be fully backed is analysed in Sargent and Wallace (1982) as the "Real Bills Doctrine".
- Here, we consider the efficiency of stationary paths. This is reasonable only if we are free to choose initial conditions. In particular, we do not consider the welfare properties of the transition path from fiat money to bank money. It is an open question whether a sequence of Pareto-improving financial innovations ( $\tau_t, \Delta_t$ ) can achieve this transition in finite time.

#### 4.4. Policy comparisons

We have looked at two quite different instruments of monetary policy: the growth rate of money supply, and the reserve ratio maintained by the banking system. We have analysed the role of each instrument separately and shown that relevant optimal policies have very different characteristics, qualitatively as well as quantitatively. One may then ask: which is a better instrument, and how do they interact when used together?

The question of fully optimal monetary policy in this framework can be phrased as the following problem

$$\text{MP}(\ast) \quad \max_{\tau > -e, r > -1} V(r, e + \tau),$$

subject to

$$\eta(r, \tau, \Delta) = 0, \quad 0 \leq \Delta \leq 1.$$

Clearly, the problems MP(1) and MP(2) are restricted versions of this, with  $\Delta=1$  or  $\tau=0$  imposed as additional restrictions. Given that this is not a convex programme, its analytical characterization or comparative statics are difficult even for our relatively simple example. This difficulty appears unavoidable in situations where individuals are heterogeneous, and monetary policy has redistributive effects. In Table 1 we report numerical

TABLE 1

*Monetary policy comparisons:  $e=1, r^*=0.06, \rho=0.1236$*

Policy		Interest Rate $r$	Transfers $\tau$	Reserves $\Delta$	Welfare exp $V$	Output $Y$
Public Debt	Theorem 3	0.000	0.000	1.000	0.61310	1.208
Optimal Inflation	Theorem 4	-0.002	0.002	1.000	0.61311	1.214
Friedman Rule	Theorem 5	0.060	0.000	0.257	0.66187	1.834
Efficient Intermediation	Theorem 6	0.080	0.000	0.000	0.68102	2.123
Full Optimum	MP(*)	0.033	0.074	0.000	0.68532	2.206

solutions for alternative financial structures, assuming  $e = 1$  and  $r^* = 0.06$ ; the latter implies  $\rho = 0.1236$ . We report the characteristics of monetary policy  $(r, \tau, \Delta)$ , the welfare criterion  $\exp(V(r, e + \tau))$ , which measures expected utility in output equivalent terms, as well as actual output  $Y = e + (1 + \rho)\bar{I}$ .

The optimal inflation rate is positive, though small, at 0.2%. It also achieves a very small welfare gain relative to zero inflation. In contrast, the Friedman Rule is welfare superior to inflationary solutions by a substantial amount—the difference is approximately 9% in output equivalent terms. Efficient intermediation requires that interest rates on money be 8%, compared to a return of 6% on capital. Finally, the solution to the full-dimensional monetary policy programme  $MP(*)$  is achieved at  $r < r^*$  and  $\tau > 0$ . At such a solution, the liquidity premium is positive, though transfers, investment, and output are large enough to compensate for the welfare loss from the lower return on money.<sup>3</sup>

We are interested in two separate questions: what is the efficient rate of return on money; and how should this be financed? The results suggest that “backed” money is a better instrument of policy in the presence of precautionary demand, so that  $V(\bar{r}, e) > V(r^*, e) > \max_r V(r, e + \tau(r))$ . There is a fairly simple intuition for this. Define  $W(r) = V(r, e + \tau(r))$ , and its first-order approximation  $W(r) \simeq W(0) + r(\partial W/\partial r)$ . Let  $\hat{r}$  be the solution to  $MP(1)$ . At an interior optimum, as in Theorem 4,  $\partial W/\partial r = 0$ , thus

$$V(\hat{r}, e + \tau(\hat{r})) \simeq W(0) = V(0, e + \tau(0)),$$

at the optimum, transfers have small, or second-order, effects on overall welfare. At the same time, the effect of reserves on  $r$  is monotonic, so that

$$V(r^*, e) \simeq W(0) + \frac{\partial V}{\partial r} r^* > W(0),$$

the Friedman Rule provides a first-order welfare improvement over the optimal inflation rate. For a similar reason, the solution to  $MP(*)$  makes for a small improvement to welfare over  $MP(2)$ .

The numerical illustration also shows the difficulty in characterizing the optimal policy with alternative instruments. The natural benchmarks for the interest rate are  $r = 0$  and  $r = r^*$ . Theorem 3 showed that  $r = 0$  is the unique stationary equilibrium with fiat money ( $\Delta = 1$ ) and inactive policy ( $\tau = 0$ ). We show in Section 5 that  $r = r^*$  is the comparable equilibrium with bank money. Even then, the solution to  $MP(1)$  yields  $r < 0$ ; the solution to  $MP(2)$  has  $r > r^*$ , while  $MP(*)$  is solved by  $0 < r < r^*$ . This property obtains for most reasonable values of  $\rho$  in the economy we study. The fact that competitive equilibrium is suboptimal is clear: the direction of inefficiency is difficult to characterize in any generality.

Lastly, there is a clear economic reason to have a higher inflation rate if money is held for precautionary, rather than transactions purposes. We have a situation where a positive proportion of the population hold excess money balances,  $m(1 + r) > \bar{C}_1$ . Idle balances are inefficient after the fact, and inflation partly taxes this inefficiency.

#### 4.5. Comments and discussion

We study a relatively simple model. Clearly, many of the results rely on its specific features. The qualitative results are robust to the specification of preferences (i.e.  $U$  and  $F$ ), and are available in Dutta and Kapur (1993). Our assumption of no aggregate risk in preferences or

3. We note that this solution requires  $I > e$ , so that the stationary policy imposes welfare costs on an initial generation.

in production is crucial. We expect that some of the qualitative results would continue to hold in the presence of production uncertainty. Clearly, the policy instruments need to be state-dependent; the desirability of the Friedman Rule is likely to be sensitive to the availability of insurance against aggregate uncertainty (see, for example, Chari, Christiano and Kehoe (1996)).

We restrict the policy problem in two ways; by the stationarity of instruments and outcomes, and by restricting policies to be non-random. The efficiency of financial innovations, and associated transition paths (as, for example, in Ireland (1994)) are of interest, and cannot be studied in the context of stationarity. Similarly, the lack of convexity in the monetary policy problem may make policy randomization desirable. Efficient paths may thus display cycles of the kind studied by Azariadis and Smith (1996) as equilibrium phenomena.<sup>4</sup>

Finally, some comments on the specific overlapping generations structure of our model. The framework of three-period overlapping generations is familiar from other papers dealing with the coexistence of money and long-lived assets (see, e.g. Dutta and Polemarchakis (1990), Bencivenga and Smith (1991), Boyd and Smith (1995)). It should also be noted that in our model money is held because of the informational asymmetry. Suppose, to the contrary, that  $I_t$  is observable at  $t+1$ . Individuals would then be able to finance consumption by selling or borrowing against their investment. The only perfect foresight stationary equilibrium path in this modified economy has  $r=r^*$ .

## 5. ON PRIVATE INTERMEDIATION

In Section 4 we looked at the efficiency properties of alternative financial structures. The framework simultaneously provides an economic role for banking. The institutional structure underlying the results on equilibria with intermediation can be understood as follows.

Banks offer an interest rate  $r$  on their deposits. Households hold some of their savings in the form of bank deposits, and make withdrawals as and when consumption needs arise. At the same time, banks invest an amount  $I_*$  of their holdings, and these investments earn the rate of return  $\rho$ . The remainder is available to finance withdrawals. The demand for bank deposits by households depends on the deposit interest rate  $r$ . If banks offer  $r \geq r^*$ , households rationally choose to hold their entire savings in the form of bank deposits.

So far we have assumed that monetary authorities can costlessly enforce the reserve ratio  $\Delta$  as well as money growth  $\tau$ . An unregulated banking system may well choose reserves and interest rates different from the optimal ones. In this section, we briefly indicate the likely direction of such deviations from efficiency.

We first consider the outcome of competitive banking. It turns out that the unique stationary equilibrium has  $r=r^*$ , i.e., corresponds to the Friedman Rule. We have already shown that this can be dominated, so that unregulated banking is likely to result in excessive reserves.

We then consider the impact of alternative forms of costly intermediation. These costs change the constraints facing society, and affect the optima as well as equilibria. In either case, the constrained efficient allocations correspond to liquidity premia being higher than

4. In an early version of this paper, Dutta and Kapur (1993), we show that cyclical equilibria arise with competitive banking.

what we obtain. We do not consider the significantly more difficult problem of decentralizing constrained optima in these environments. The properties of optimal policy are likely to be sensitive to the exact specification of such constraints. We discuss this issue briefly below, leaving further analysis for future research.

Direct costs of intermediation are incorporated quite easily in the analysis. These may arise from alternative sources, such as costly monitoring, record keeping, or verification of output from projects. Lack of enforcement may impose an indirect cost, as follows. Bankers may abscond with investment income and declare a bank failure, unless the value of the bank as a going concern is sufficiently large. Such an outcome can only be sustained if banks make profits, and this by paying low enough interest rates on deposits. Indeed, interest rates must be strictly lower than  $r^*$ .

5.1. *Competitive banking*

Consider the profits of a bank of fixed size  $\sigma$ , which has access to depositors with endowment  $\sigma e$  at each  $t$ . At each time  $t$ , deposits are  $D_t = \sigma m_t$ ; total withdrawals are  $X_t = \sigma(\bar{C}_t + \bar{C}_{2t} - (1 + \rho)I_{t-2})$ . In addition, the bank receives investment income  $(1 + \rho)I_{*,t-2}$ , and chooses current investment  $I_{*,t}$ . Profits in period  $t$  are

$$v_t = D_t - X_t + (1 + \rho)I_{*,t-2} - I_{*,t}.$$

Given a sequence of interest rates  $\{r_{t+i}; i = 0, 1, 2, \dots\}$ , the present discounted value of profits is

$$B_t = \sum_{i=0}^{\infty} \frac{v_{t+i}}{\prod_{j=0}^i (1 + r_{t+j})}.$$

A competitive bank takes the entire sequence of interest rates  $\{r_{t+i}; i \geq 0\}$  as given, and chooses  $I_{*,t}$  to maximize  $B_t$  at each  $t$ , subject to the constraint

$$0 \leq I_{*,t} \leq (1 + \rho)I_{*,t-2} + D_t - X_t.$$

A competitive equilibrium with private banking is a sequence  $\{I_{*,t}, r_t\}$  such that banks maximize profits given  $\{r_t\}$ , and goods and money markets clear at each  $t$ .

As before, we consider stationary paths  $r_t = r$ . The quantity of net deposits is stationary and equal to  $Z(r) = e((1 + \rho) - C_1(r) - C_2(r) - \rho\mu(r))$  in the aggregate, where  $C_i = \bar{C}_i/e$ . The present value of profits for a bank of size  $\sigma$  depends on the bank's investment choices,  $I_{*,i}$ , for  $i \geq -2$ ,

$$B_{\sigma,t}(\{I_{*,i}\}, r) = (1 + \rho)I_{*,t-2} + \left(\frac{1 + \rho}{1 + r}\right)I_{*,t-1} + \left(\frac{1 + \rho}{(1 + r)^2} - 1\right) \sum_{i=0}^{\infty} \frac{I_{*,t+i}}{(1 + r)^i} + \left(\frac{1 + \rho}{r}\right) \sigma Z(r).$$

The first two terms denote the discounted value of income from past investments; the third is the discounted sum of net income from current and future investments, and the last is the present value of the stream of net deposits.

**Theorem 7** (Competitive banking). *Define  $\Delta^* = (\rho - r^*)/2\rho$  and let  $\tau = 0$ . The stationary competitive equilibrium with private banking is unique, and has  $r = r^*$ ,  $I_* = (1 - \Delta^*)e$ . Banks make zero profits at each  $t$ :  $v_t = B_t = 0$ .*



We know, from Theorems 5 and 6, that the Friedman Rule can be dominated: thus, competitive banking fails to attain the efficient level of intermediation whenever  $\rho > 0$ . Indeed, at any  $r > r^*$ , a competitive bank would choose to invest nothing. Active monetary policy can improve upon this outcome if the planner takes account of the effect of the reserve ratio on interest rates, which competitive banks do not. We also note that a legal restriction on minimal reserves will bound interest rates away from  $\bar{r}$ ; it may nevertheless be compatible with the Friedman Rule whenever this minimum is less than  $\Delta^*$ .

### 5.2. On costly intermediation

We have assumed that intermediation is costless, which is unlikely to be well-founded. Costly intermediation could force the liquidity premium to be positive, even at constrained optima.

Our informational restriction is extreme; it leads to the closure of secondary capital markets, and, at the same time, it allows intermediation to be costless. A natural modification is that investment is observable at a cost (as in Townsend (1979)). Costly verification would impose upper bounds on  $\gamma$ . Fixed costs, or other forms of returns to scale, as in Diamond (1984), would make competitive banking unsustainable.

If intermediation is costly, efficiency requires that it be done at the lowest possible cost. It is useful to note that the Friedman Rule may continue to be feasible if intermediation costs are not too high. Suppose  $\phi(I_*)$  is the cost of intermediating investment  $I_*$ . The Friedman Rule  $r = r^*$  is feasible whenever

$$C_1(r^*) + C_2(r^*) \leq (1 + \rho) - \frac{\phi(e)}{e} \Rightarrow \frac{\phi(e)}{e} \leq \frac{\rho - r^*}{2}.$$

Using the approximation  $r^* \simeq \rho/2$ , this inequality holds if the average cost of intermediation does not exceed 25% of its return.

### 5.3. On incentive-compatible intermediation

A somewhat different problem arises when we consider the motives of intermediaries. Even if intermediation is costless, enforcement problems may make the zero profit outcome unsustainable. Shareholders in privately held banks would find it both feasible, and desirable, to run away with the money when depositors want to make withdrawals.

The shareholders of a bank can declare a failure at any time  $t$ , and pay out current investment income as dividends. The only penalty is that the bank must cease all further operations at that time. The value of declaring default at  $t$  is current investment income, which shareholders can appropriate: this is just  $(1 + \rho)I_{*,t-2}$ . The value of the bank as a going concern depends on the entire sequence of investments and interest rates: this is  $B_t(\{I_{*,i}, r_i\})$ , as defined earlier. A path of investments and interest rates  $\{I_{*,i}, r_i\}$  is incentive-compatible only if  $B_t(\{I_{*,i}, r_i\}) \geq (1 + \rho)I_{*,t-2}$  at each  $t$ . This restriction—a straightforward application of the “temporary incentive compatibility” condition of Green (1987) and Atkeson and Lucas (1992)—ensures that it is in the interest of any bank to continue to meet its obligations at each time. It is simple to check that the Friedman Rule is not incentive-compatible, as  $B(I_*, r^*) = 0$ , and  $I_* > 0$ .

The precise regulations for controlling intermediaries will clearly affect the class of sustainable solutions. A positive theory of incentive-compatible interest rates is not our

main concern here.<sup>5</sup> Alternative models studied by Farmer (1988), Boyd and Smith (1995), and Azariadis and Smith (1996) establish that monetary policy can have complicated, and possibly perverse, effects in the presence of asymmetric information in financial markets, and of non-linear contracts designed to cope with such asymmetries. In a similar vein, we have little to say about the efficient regulation of the banking system in the presence of enforcement problems. Our main point, that monetary policy needs to be studied with a richer class of policy instruments, remains valid in situations where multilateral private information affects markets for intertemporal trade.

## 6. CONCLUSIONS

This paper examined liquidity preference and the precautionary demand for money in a simple model. The notion of liquidity makes sense if there are two simultaneous problems affecting asset trade: that consumers face some uninsured individual risks that makes their consumption plans unpredictable; and that some assets are more likely to be subject to asymmetric information. The first could, in itself, be a consequence of private information, as pointed out by Green (1987) and Atkeson and Lucas (1992), among others. As to the second, our assumption is extreme in that it leads to a complete breakdown in the market for one type of asset. It is possible to deal with more realistic versions.

We use the model to analyse some basic issues about the expansionary effects of inflation. Quite importantly, such expansion is often welfare-inferior. Further, the positive association between inflation and output is likely to break down as the financial system becomes more sophisticated, for example, as the liquidity services of fiat money are superseded by that of interest-bearing bank deposits.

This analysis has something to say about a few "anomalies" often remarked upon in empirical studies. The first issue concerns the dependence of aggregate output on the nominal interest rate (Litterman and Weiss (1985)). The association is often explained in the context of models where money is held for transactions purposes, as in Grossman and Weiss (1983) and Lucas and Stokey (1987). The precautionary motive, which we focus on, allows some evaluation of alternatives to cash as a liquid asset. The second issue is the "equity premium" puzzle (Mehra and Prescott (1985)). This is often associated with incomplete markets, or with restricted participation by consumers in equity markets. Our analysis suggests that an approach based on the precautionary demand for riskless assets may be fruitful in understanding both issues in the same framework. In the absence of intermediation, the return on liquid assets is equal to the growth rate of population, independently of preferences and technology. At first pass, it offers an explanation of why the rate of return on short term bonds is so low, which appears to lie at the heart of the puzzle (Weil (1989)). We studied a model with no production uncertainty, so that the issue of sustainable equity premia cannot be analysed directly. A third stylized fact arises from studies of consumption behaviour, (see, e.g. Hall (1989)): the consumption behaviour of households is liquidity-constrained. We emphasize that this is endogenous.

These facts may well arise from the same source. The source is that asset markets are incomplete, that this incompleteness is likely to affect the ability of households to insure against individual risks, and that the available set of assets may be restricted in liquidity as well as risk characteristics. Such market restrictions are likely to constrain the ability of households to smooth consumption over time and across states of nature. The incidence of liquidity constraints, and the size of the liquidity premium, are closely related. In order

5. We have done so elsewhere and in a wider class of environments: see Dutta and Kapur (1997).

to understand the determination of the liquidity premium in an actual economy, the presence of financial intermediation should be taken into account. It remains to be seen whether the equity premium can be understood as a liquidity premium once actual measures of the degree of intermediation are accounted for.

This paper provides a relatively simple framework to judge the impact of liquidity constraints on individuals facing different types of markets for liquidity. The transmission mechanism for financial policies obviously depends on the nature of constraints which make the policy effective in the first place. Our results suggest that it is just as sensitive to the assets and institutions which mediate these constraints. Boyd and Smith (1995) and Azariadis and Smith (1996) analyse the properties of intertemporal equilibria and the effects of monetary policy in models where financial contracts must be incentive-compatible. The differential role of banks in the financing of firms in different countries has provoked much recent interest (e.g. Hellwig (1991)). It would be useful to know whether such institutional differences make for substantive changes in the quantitative or temporal properties of the monetary transmission mechanism, as this theoretical work suggests.

## APPENDIX

### *Proof of Theorem 1*

For  $\gamma = 1 - ((1+r)^2/(1+\rho)) < 1$ , the individual solves

$$(CP) \quad \max_{m, c_1(\theta)} \int_0^1 [\theta \ln c_1(\theta) + (1-\theta) \ln ((1+\rho)(w-\gamma m) - (1+r)c_1(\theta))] d\theta,$$

subject to

$$c_1(\theta) \leq m(1+r), \quad (1)$$

$$0 \leq m \leq w. \quad (2)$$

We solve (CP) separately for the cases  $\gamma \leq 0$  and  $\gamma \in (0, 1)$ . The solution for the first case is straightforward. If  $\gamma \leq 0$ , the objective function is nondecreasing in  $m$ , and the constraint set  $[0, m(1+r)]$  for  $c_1$  is strictly increasing. It is then optimal to choose  $m = w$ ; if we define  $\mu = m/w$ , the optimal  $\mu = 1$ . Consumption is not liquidity constrained:  $c_1 = w\theta(1+r)$  for each  $\theta$ .

The proof for  $\gamma \in (0, 1)$  is in two steps. The first step considers the choice of  $c_1(\theta)$  for a given  $m$ , and the second solves for optimal  $m$ . For  $m \in [0, w]$ , define  $y(m) = w - \gamma m$ . For each realization of  $\theta \in [0, 1]$ , the Lagrangean for the choice of  $c_1(\theta)$  writes as

$$\mathcal{L}(c_1, \phi; m, \theta) = \theta \ln c_1 + (1-\theta) \ln ((1+\rho)y(m) - (1+r)c_1) + \phi(m(1+r) - c_1),$$

where  $\phi$  is the multiplier for the liquidity constraint (1). The first order conditions are

$$\frac{\theta}{c_1} - \frac{(1-\theta)(1+r)}{(1+\rho)y(m) - (1+r)c_1} - \phi = 0, \quad (3)$$

$$\phi(m(1+r) - c_1) = 0, \quad (4)$$

and

$$(1+r)m - c_1 \geq 0, \quad \phi \geq 0. \quad (5)$$

Define

$$\tilde{c}_1(m, \theta) = \theta \frac{(1+\rho)y(m)}{1+r}, \quad \tilde{c}_2(m, \theta) = (1-\theta)(1+\rho)y(m). \quad (6)$$



Note that  $\tilde{c}_1(m, \theta)$  solves the unconstrained problem  $\mathcal{L}(c_1, 0; m, \theta)$  for each  $\theta \in [0, 1]$ . The solution to the constrained optimization problem  $\mathcal{L}(c_1, \phi; m, \theta)$  is

$$c_1 = \tilde{c}_1(m, \theta); \quad \phi = 0 \quad \text{if } \tilde{c}_1(m, \theta) \leq m(1+r), \tag{7}$$

$$c_1 = m(1+r); \quad \phi > 0 \quad \text{otherwise.} \tag{8}$$

As  $\tilde{c}_1(m, \theta)$  is increasing in  $\theta$ , we have

$$\tilde{c}_1(m, \theta) \leq m(1+r) \Leftrightarrow \theta \leq \theta(m) \equiv \frac{m(1-\gamma)}{y(m)},$$

consumption is liquidity constrained if and only if  $\theta > \theta(m)$ . Define

$$\tilde{V}(m; \theta) = \theta \ln \tilde{c}_1(m, \theta) + (1-\theta) \ln \tilde{c}_2(m, \theta) \tag{9}$$

$$\hat{V}(m; \theta) = \theta \ln (m(1+r)) + (1-\theta) \ln ((1+\rho)(w-m)). \tag{10}$$

The indirect utility of carrying money  $m$ , with preference shock  $\theta$ , is

$$V(m; \theta) = \begin{cases} \tilde{V}(m; \theta) & \text{if } \theta \leq \theta(m) \\ \hat{V}(m; \theta) & \text{if } \theta > \theta(m). \end{cases} \tag{11}$$

Using the uniform distribution, the expected utility of holding  $m$  writes as

$$V(m) = E_\theta V(m; \theta) = \int_0^{\theta(m)} \tilde{V}(m; \theta) d\theta + \int_{\theta(m)}^1 \hat{V}(m; \theta) d\theta.$$

We need two properties of  $V(m)$ .

**Result 1.** Let  $\theta(m) = m(1-\gamma)/y(m)$ . For each  $m \in (0, w)$ ,

$$\tilde{V}(m; \theta(m)) = \hat{V}(m; \theta(m)) \quad \text{and} \quad \left. \frac{\partial \tilde{V}}{\partial m} \right|_{\theta(m)} = \left. \frac{\partial \hat{V}}{\partial m} \right|_{\theta(m)}.$$

*Proof.* The first assertion is obvious and follows from the definitions. To verify the second, note that

$$\frac{\partial \tilde{V}}{\partial m} = -\frac{\gamma}{y(m)} \quad \text{and} \quad \frac{\partial \hat{V}}{\partial m} = \frac{\theta}{m} - \frac{1-\theta}{(w-m)}.$$

Evaluating the latter at  $\theta(m)$  completes the proof.  $\parallel$

**Result 2.**  $V(m)$  is strictly concave in  $m \in (0, w)$ .

*Proof.* Note that

$$\frac{\partial^2 \tilde{V}}{\partial m^2} = -\frac{\gamma^2}{y^2(m)} < 0 \quad \text{and} \quad \frac{\partial^2 \hat{V}}{\partial m^2} = \frac{-\theta}{m^2} - \frac{1-\theta}{(w-m)^2} < 0.$$

Since  $V(m)$  is a convex combination of  $\tilde{V}(m, \theta)$  and  $\hat{V}(m, \theta)$ , it must be strictly concave.  $\parallel$

Consider the problem

$$\max_{0 \leq m \leq w} V(m). \tag{12}$$

We know, from Results 1 and 2, that  $V(m)$  is continuously differentiable and strictly concave. The first-order condition for an interior maximum is

$$\frac{\partial V(m)}{\partial m} = \int_0^{\theta(m)} \frac{\partial \tilde{V}(m, \theta)}{\partial m} d\theta + \int_{\theta(m)}^1 \frac{\partial \hat{V}(m, \theta)}{\partial m} d\theta = 0.$$

Using the facts that

$$\frac{\partial \tilde{V}}{\partial m} = -\frac{\gamma}{y(m)} \quad \text{and} \quad \frac{\partial \hat{V}}{\partial m} = \frac{\theta w - m}{m(w-m)},$$

and integrating over the specified limits, we obtain

$$\frac{-\gamma}{y(m)} \theta(m) + \frac{1 - (\theta(m))^2}{2m} - \frac{(1 - \theta(m))^2}{2(w - m)} = 0.$$

Finally, using  $\theta(m) = m(1 - \gamma)/y(m)$ , and hence  $1 - \theta(m) = (w - m)/y(m)$ , we have

$$2m^2\gamma^2 - (1 + 2\gamma)mw + w^2 = 0, \quad (13)$$

or

$$2\gamma^2\mu^2 - (1 + 2\gamma)\mu + 1 = 0,$$

where  $\mu \equiv m/w$ , as before. The second-order condition is satisfied at the lower root,

$$\mu = \frac{1 + 2\gamma - \sqrt{1 + 4\gamma - 4\gamma^2}}{4\gamma^2}. \quad (14)$$

The optimal portfolio holds a proportion  $\mu(\gamma)$  of total wealth as money, and the rest is invested. Given optimal money holdings  $m = \mu w$ , consumption is given by equations (6) to (8). This completes the proof of Theorem 1.  $\parallel$

*Proof of Property 1.* The value  $\mu(1) = \frac{1}{2}$  obtains by substitution in equation (14). We can also check, using L'Hospital's rule, that  $\lim_{\gamma \rightarrow 0} \mu(\gamma) = 1 = \mu(0)$ , so that the function is continuous at  $\gamma = 0$ . Differentiation of (14) yields

$$\frac{d\mu(\gamma)}{d\gamma} = \frac{1}{2\gamma^3\sqrt{1 + 4\gamma(1 - \gamma)}} (1 + 3\gamma - 2\gamma^2 - (1 + \gamma)\sqrt{1 + 4\gamma(1 - \gamma)}).$$

It is straightforward to check that this expression is negative for  $\gamma \in (0, 1)$ .  $\parallel$

Result 3 is useful for the next property and further proofs.

**Result 3.** For  $\gamma \in (0, 1)$ , there exists a threshold value of impatience,  $\theta_*(\gamma) \in (0, 1]$  such that an individual is liquidity constrained whenever  $\theta > \theta_*(\gamma)$ . We have

$$\theta_*(\gamma) = \frac{\mu(1 - \gamma)}{1 - \gamma\mu} = 1 - 2\gamma\mu(\gamma) = \sqrt{2\mu(\gamma) - 1}. \quad (15)$$

This threshold  $\theta_*(\gamma) < \mu(\gamma)$  for  $\gamma \in (0, 1)$ . Further,  $\theta_*(\gamma)$  is decreasing in  $\gamma$ , with  $\theta_*(0) = 1$ , and  $\theta_*(1) = 0$ .

*Proof.* From the proof of Theorem 1, the threshold value of impatience is  $\theta(m) = m(1 - \gamma)/y(m)$ . At the optimal portfolio  $m = \mu w$ , so that  $\theta_*(\gamma) = \mu(1 - \gamma)/(1 - \gamma\mu)$ . That proves the first equality in (15). Substitution of (13) yields

$$\mu(1 - \gamma) = (1 - \mu\gamma)(1 - 2\mu\gamma),$$

proving the second equality in (15), and also

$$\sqrt{2\mu - 1} = 1 - 2\gamma\mu,$$

proving the last equality. Further,  $\mu^2 > 2\mu - 1$  whenever  $\mu < 1$ . Substitution from Property 1 completes the proof.  $\parallel$

*Proof of Property 2.* Follows from the fact that  $\lambda(\gamma) = \text{Prob}(\theta \geq \theta_*(\gamma)) = 1 - \theta_*(\gamma)$  and Result 3.  $\parallel$

*Proof of Theorem 2*

For  $\gamma \in (0, 1)$ , the ex ante expected utility is

$$V(r; w) = \int_0^{\theta_*(\gamma)} \tilde{V}(\mu w; \theta) d\theta + \int_{\theta_*(\gamma)}^1 \hat{V}(\mu w; \theta) d\theta,$$

where  $\theta_*(\gamma)$  is as defined in Result 3, and

$$\tilde{V}(\mu w; \theta) = \theta \ln \theta + (1 - \theta) \ln(1 - \theta) + \ln((1 + \rho)w(1 - \mu\gamma)) - \theta \ln(1 + r),$$

$$\hat{V}(\mu w; \theta) = \theta \ln(\mu w(1 + r)) + (1 - \theta) \ln((1 + \rho)w(1 - \mu)).$$

Evaluating the integral, noting that  $\int x \ln x dx = (x^2/2)(\ln x - \frac{1}{2})$ , and  $\lim_{x \rightarrow 0} x^2 \ln x = 0$  by L'Hospital's Rule, we obtain

$$V(r; w) = \ln w - \frac{1}{2}(\theta_*(\gamma) - \ln(1 + \rho)(1 + r)\mu(1 - \gamma\mu)).$$

Substitution from (15) completes the proof. For  $\gamma \leq 0$ , all wealth is held as money and consumption is not liquidity constrained. The result obtains by evaluating the integral

$$V(r; w) = \int_0^1 [\theta \ln \theta + (1 - \theta) \ln(1 - \theta)] d\theta + \ln w + \frac{3}{2} \ln(1 + r). \quad \parallel$$

*Proof of Property 3.* We know, from the envelope theorem, that  $\partial V / \partial \mu|_{\mu(\gamma)} = 0$ . Using this

$$\frac{\partial V}{\partial r} = \frac{1 + 2\theta_*^2(\gamma)}{2(1 + r)}, \tag{16}$$

where  $\theta_*(\gamma) \equiv 1$  for  $\gamma \leq 0$ . This is positive whenever  $r > -1$ .  $\parallel$

Result 4 collects some useful properties of the function  $\eta(r, \tau, \Delta)$  in the market clearing relation (E).

**Result 4.** Let  $\tau > -e, r > -1$ , and  $\Delta \in [0, 1]$ ; define  $1 + r^* = \sqrt{1 + \rho}$ . The function  $\eta(r, \tau, \Delta)$  has the following properties:

1.  $\eta(r, 0, 1) = 0$  if and only if  $r = 0$ ;
2.  $\eta(r, \tau, 1)$  is increasing in  $r$ , and in  $\tau$ ;
3.  $\eta(r^*, 0, \Delta) = 0$  if and only if  $\Delta = (\rho - r^*) / 2\rho$ ;
4.  $\eta(r, 0, \Delta)$  is increasing in  $\Delta$ , and in  $r$  for  $r \geq r^*$ . Let  $\bar{r} = (\sqrt{9 + 8\rho} - 3) / 2 > r^*$ .  $\eta(r, 0, 0) = 0$  if, and only if,  $r = \bar{r}$ . Further,  $\bar{r} = \max \{r : \eta(r, 0, \Delta) = 0, \Delta \in [0, 1]\}$ .

*Proof.* We use Theorem 1. For  $r \geq r^*$ , individuals hold only money and aggregate consumption in period 1 is  $\int_0^1 \theta w(1 + r) d\theta$ . For  $r < r^*$ , the liquidity premium is positive and aggregate consumption is  $\int_0^1 \min[\tilde{c}_1(\theta), m(1 + r)] d\theta$ . Evaluating these integrals

$$\tilde{C}_1 = \begin{cases} w[\frac{1}{2}\mu(1 + r)(2 - \theta_*(r))] & \text{if } r < r^*; \\ w((1 + r)/2) & \text{if } r \geq r^*. \end{cases}$$

Similar calculations show that aggregate consumption in period 2 is

$$\bar{C}_2 = \begin{cases} w[(1 + \rho)(1 - \mu + \frac{1}{2}\mu(1 - \gamma)\theta_*(r))] & \text{if } r < r^*; \\ w((1 + r)^2/2) & \text{if } r \geq r^*. \end{cases}$$

Define  $C_i(r) = \bar{C}_i/w$ , and  $\alpha(r) = C_1(r) + C_2(r) - \rho(1 - \mu(r))$ . We write

$$\alpha(r) = \begin{cases} \alpha_0(r) & \text{if } r < r^*; \\ \alpha_1(r) & \text{if } r \geq r^*, \end{cases}$$

where

$$\alpha_0(r) = 1 + \mu(r)r(1 + \frac{1}{2}(1 + r)\theta_*(r)),$$

$$\alpha_1(r) = \frac{(1 + r)(2 + r)}{2},$$

and  $\alpha_0(r^*) = \alpha_1(r^*)$ . The market clearing condition (E) is

$$\eta(r, \tau, \Delta) \equiv (e + \tau)(\alpha(r) - \rho(1 - \Delta)\mu(r)) - e = 0.$$

1. Note that  $\eta(r, 0, 1) = e(\alpha(r) - 1)$ . Since  $\alpha(0) = 1$ , we have  $\eta(0, 0, 1) = 0$ . Also,  $\alpha(r) > 1$  for  $r \geq r^*$ , so that  $\eta(r, 0, 1) = 0$  and  $\mu(r) > 0$  imply either  $r = 0$  or  $2 + (1 + r)\theta_*(r) = 0$ . The latter is impossible as  $r > -1$  and  $\theta_* \geq 0$ .
2. Next, note that  $\eta(r, \tau, 1) = (e + \tau)\alpha(r) - e$ , and  $\alpha(r)$  is positive, so  $\eta$  is increasing in  $\tau$ . Given  $\tau > -e$ , the function is increasing in  $r$  whenever  $\alpha(r)$  is increasing in  $r$ : that  $\alpha_1(r)$  is increasing is obvious; that  $\alpha_0(r)$  is increasing is implied by Property 1 and Result 3.

3.  $\eta(r^*, 0, \Delta) = 0 \Leftrightarrow \Delta = (1 + \rho - \alpha_1(r^*)) / \rho = (\rho - r^*) / 2\rho$ . Note that  $\rho > r^* > 0$  for  $\rho > 0$ , so that  $(\rho - r^*) / 2\rho \in (0, 1)$ .
4. We have  $\eta(r, 0, \Delta) = e(\alpha(r) - \rho(1 - \Delta)\mu(r))$ , which is increasing in  $\Delta$ . For  $r \geq r^*$ , we have  $\eta(r, 0, \Delta) = e(\alpha_1(r) - \rho(1 - \Delta))$ , which is increasing in  $r$ . It follows that

$$\eta(\bar{r}, 0, 0) = 0 \Rightarrow \bar{r} = \max \{r : \eta(r, 0, \Delta) = 0; \Delta \in [0, 1]\}.$$

This yields  $(1 + \bar{r})(2 + \bar{r}) = 2(1 + \rho)$ : the larger root is  $\bar{r} = (\sqrt{9 + 8\rho} - 3) / 2 > r^*$ .

#### Proof of Theorem 3

Statement 1 follows from Result 4.1 above. Statement 2 follows from the fact that  $\bar{I} = I = e(1 - \mu(\gamma))$  whenever  $\tau = 0$  and  $\Delta = 1$ ; further,  $\gamma = \rho / (1 + \rho) > 0 \Rightarrow \mu(\gamma) < 1$ . Finally,  $\lambda(\gamma) = 2\gamma\mu(\gamma) > 0$  whenever  $\gamma > 0$ . This proves Statement 3.  $\parallel$

#### Proof of Theorem 4

For the specified range of  $\tau$ , the existence of an equilibrium  $r(\tau)$  follows from Result 4.2. That  $r(\tau)$  is decreasing in  $\tau$  follows from the fact that  $\alpha_0(r)$  is increasing both in  $r$  and in  $\tau$ . If so, inflation  $\pi = (-r(\tau)) / (1 + r(\tau))$  is increasing in  $\tau$ .

Aggregate output is  $Y = e + (1 + \rho)(1 - \mu(r))(e + \tau)$ . If  $r$  is decreasing in  $\tau$ , and  $\mu(r)$  increasing in  $r$  by Property 1, it follows that  $Y$  is increasing in  $\tau$ .

Statements 3 and 4 characterize the solution to **MP(1)**. Define  $W(r) = V(r; w = e/\alpha(r))$ ; the problem **MP(1)** is equivalent to  $\max_{r > -1} W(r)$ . It is straightforward to check that  $W(r)$  is continuous at  $r = r^*$ , and decreasing in  $r$  for  $r > r^*$ . Hence, whenever a maximum,  $\hat{r}$ , exists, we must have  $\hat{r} \leq r^*$ , so that

$$\max_{-1 < r} W(r) = \max_{-1 < r \leq r^*} W(r) = W(\hat{r}).$$

Let  $\mathcal{R} = [-1, r^*]$ . Using Theorem 2, we have for  $r \in \mathcal{R}$

$$W(r) = \ln(e) - \ln(\alpha_0(r)) - \frac{1}{2}(\theta_*(r) - \ln((1 + \rho)(1 + r)\mu(r)(1 - \gamma\mu(r)))).$$

$\mathcal{R}$  is compact, and  $W(r)$  is continuous on  $\mathcal{R}$ . An interior maximum exists whenever  $W(r)$  is increasing at the lower boundary and decreasing at the upper boundary, simultaneously. From equation (16) and Result 3, we have

$$\frac{\partial W}{\partial r} = \frac{2\mu(r) - \frac{1}{2}}{1 + r} - \frac{\partial \ln \alpha_0}{\partial r}, \quad \text{for } r \in \mathcal{R}. \quad (17)$$

Evaluating  $\partial \ln \alpha_0 / \partial r = \alpha'_0 / \alpha_0$ , we obtain

$$\left. \frac{\partial \ln \alpha_0}{\partial r} \right|_{r=-1} = 1 \quad \text{and} \quad \left. \frac{\partial \ln \alpha_0}{\partial r} \right|_{r=r^*} = \frac{3 + 21r^* + 10r^{*2}}{(1 + r^*)^2(2 + r^*)} > \frac{2\mu(r^*) - \frac{1}{2}}{1 + r^*}.$$

Substitution in (17) implies

$$\lim_{r \rightarrow -1} \frac{\partial W}{\partial r} = \infty; \quad \text{and} \quad \left. \frac{\partial W}{\partial r} \right|_{r=r^*} < 0.$$

Thus,  $-1 < \hat{r} < r^*$  as claimed. That  $\hat{r} > -1$  implies that  $\hat{\pi}$  is finite. This proves Statement 3.

The function  $W(r)$  is differentiable in  $(-1, r^*]$ . An interior maximum  $\hat{r}$  must satisfy the first order condition  $(\partial W / \partial r)|_{r=\hat{r}} = 0$ . Note that

$$\left. \frac{\partial \ln \alpha_0}{\partial r} \right|_{r=0} = \mu(1 + \frac{1}{2}\theta_*). \quad (18)$$

Substitution in (17), at  $r = 0$ , yields

$$\left. \frac{\partial W}{\partial r} \right|_{r=0} = \frac{-\theta_*(\mu - \theta_*)}{2} \Big|_{r=0} < 0. \quad (19)$$

Hence  $\hat{r} \neq 0$ , and  $W(r)$  is increasing in  $\tau$  at 0, as claimed in Statement 4.  $\parallel$

*Proof of Theorem 5*

We show that a solution with  $r=r^*$ ,  $\tau=0$ , and  $\Delta \in [0, 1]$  exists. From Result 4.3, this is true for  $\Delta = (\rho - r^*) / 2\rho > 0$ . From Theorem 1,  $\mu = 1$  whenever  $r \geq r^*$ , implying  $I=0$  and  $\delta=1$ . From Property 2,  $\lambda=0$  whenever  $r = r^*$ . ||

*Proof of Theorem 6*

From Property 3,  $V(r, e)$  is monotonically increasing in  $r$ . The solution to MP(2) corresponds to  $r = \max \{r: \eta(r, 0, \Delta) = 0; \Delta \in [0, 1]\}$ . From Result 4.4, this is equal to  $\bar{r}$ , with  $\Delta=0$ . Further,  $\bar{r} > r^* \Rightarrow \gamma < 0$ . From Theorem 1,  $\mu(\bar{r})=1$ , and  $I=0$ : hence  $\delta=1$ . From the definitions,  $I_* = \mu(1-\Delta)e = e$ , and  $Y = e + (1+\rho)I_* = (2+\rho)e$ . ||

*Proof of Theorem 7*

Let  $r_t = r$  for each  $t$ . The present value of a bank is  $B_t(\{I_{*,t}\}, r)$ . Consider the choice of  $I_{*,t}$  at  $t$ .

Suppose  $r > r^*$ ; clearly,  $B_t$  is monotonically decreasing in  $I_{*,t}$ , so that  $I_{*,t} = 0$  for each bank. This implies  $\Delta = 1$  in the aggregate. The stationary interest rate  $r$  satisfies  $\eta(r, 0, 1) = 0$ ; from Result 4.1,  $r = 0 < r^*$ , implying a contradiction.

Suppose instead that  $r < r^*$ .  $B_t$  is monotonically increasing in  $I_{*,t}$ , so that  $I_{*,t} = (1+\rho)I_{*,t-2} + \sigma Z(r)$  for a bank of size  $\sigma$ . The stationary path  $I_*$  satisfying this at each  $t$  yields  $\rho I_* = e(C_1(r) + C_2(r) - \rho\mu(r) - \rho - 1)$  in the aggregate. This is compatible with restriction (E) if, and only if,  $\mu(r) = 1 \Rightarrow r \geq r^*$  which is a contradiction.

It follows that  $r = r^*$  is the only candidate for a stationary equilibrium. From Theorem 5, this is feasible, and corresponds to  $\Delta^* \in (0, 1) \Rightarrow (1+\rho)I_* + \sigma Z(r) \geq I_* > 0$ .

Finally, let  $I_* = \sigma(1-\Delta^*)$  for a bank of size  $\sigma$ , and note that  $v_t = \rho I_* + \sigma Z(r) = \sigma \eta(r^*, 0, \Delta^*) = 0$ . It follows that  $B_t = 0$  at each  $r$ . ||

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